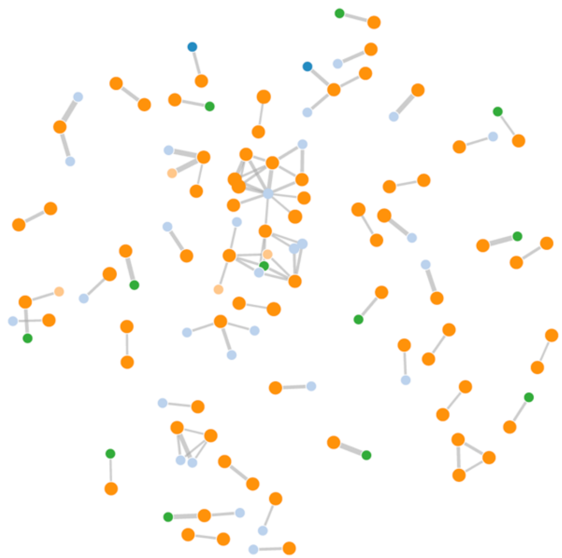
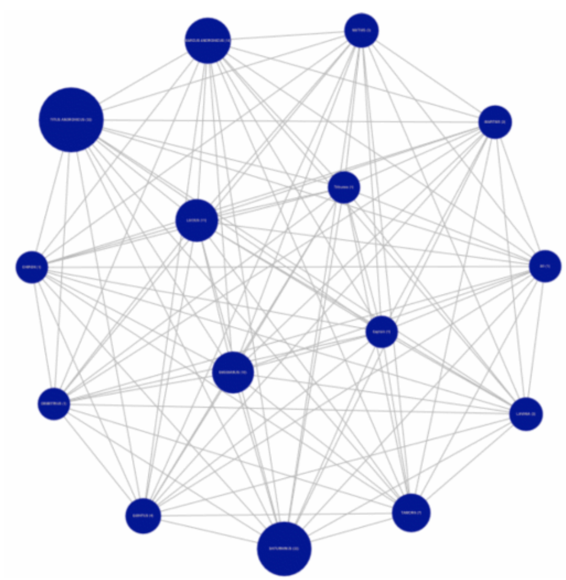
Today’s announcements:

Final exam 12/14, 7-10p, Locations TBA. Format…

Exam Reviews -





How would you characterize the difference between these graphs?

Prim's algorithms (1957) is based on the Partition Property:

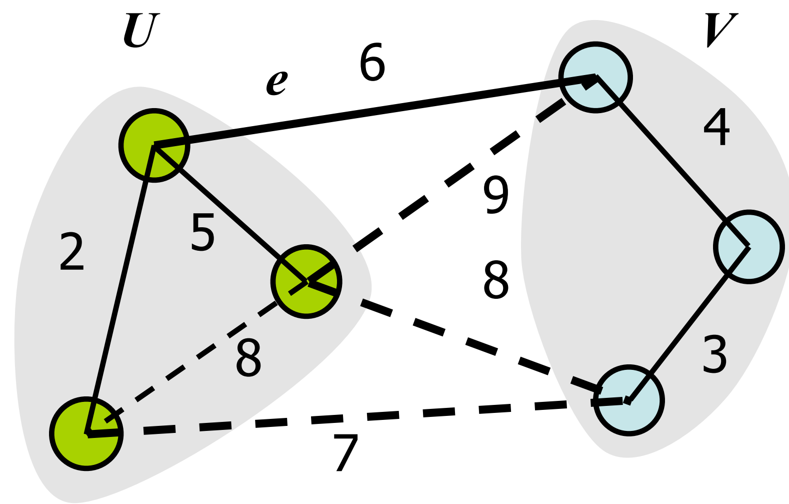
Consider a partition of the vertices of G into subsets U and V.

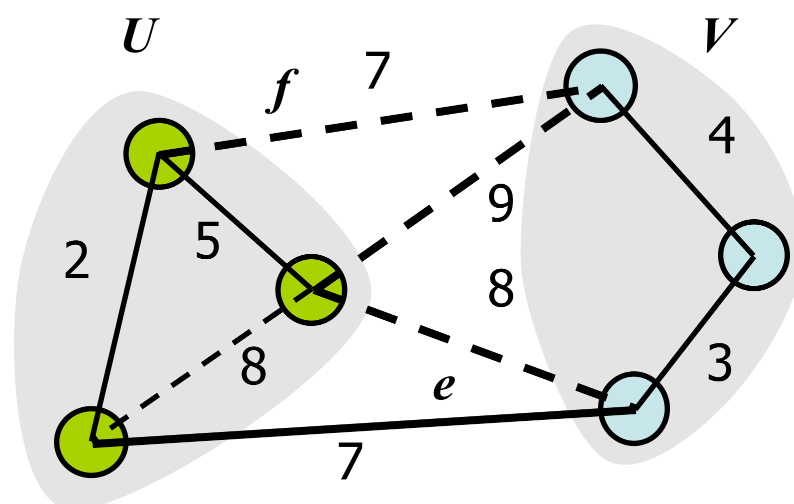
Let e be an edge of minimum weight across the partition.

Then e is part of some minimum spanning tree.

Proof:

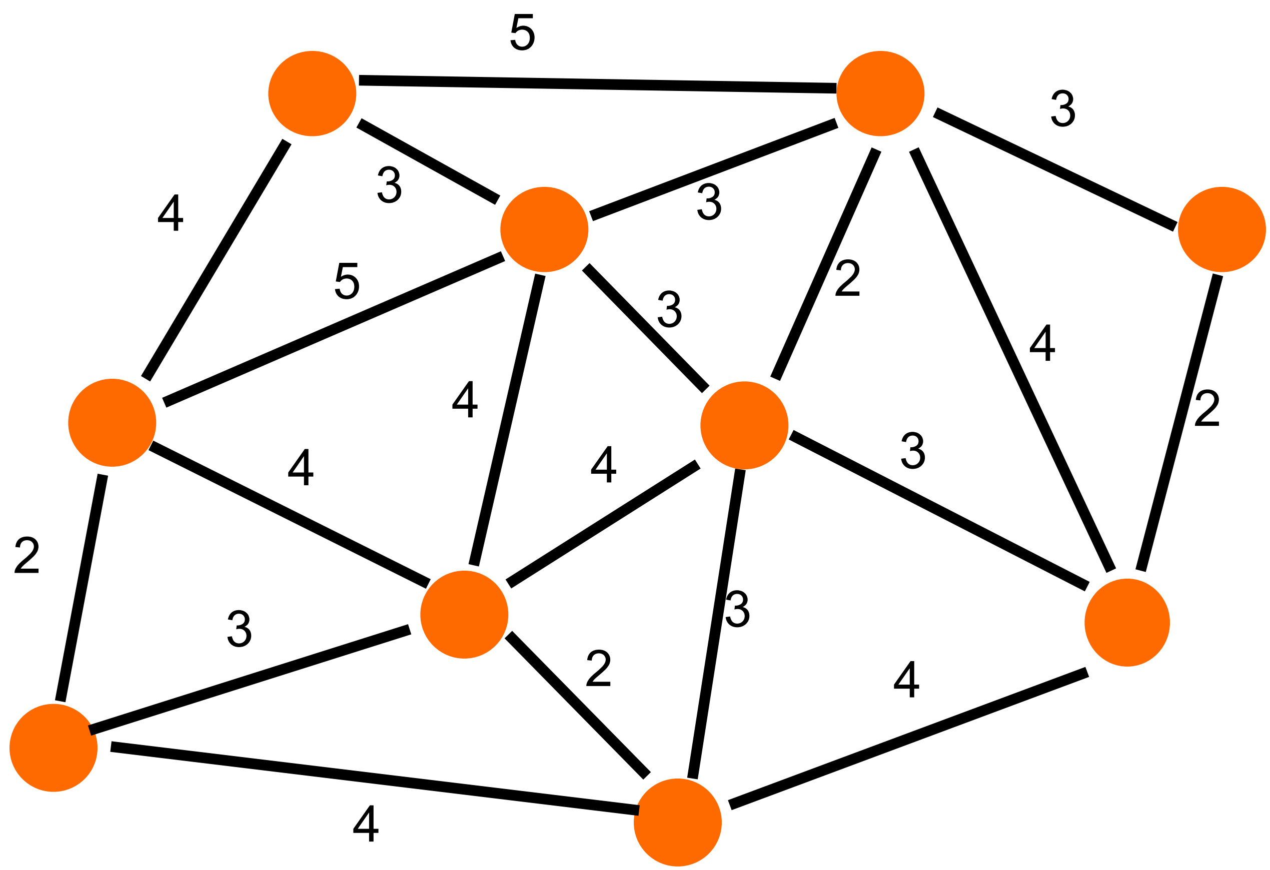
See cs374





MST - minimum total weight spanning tree

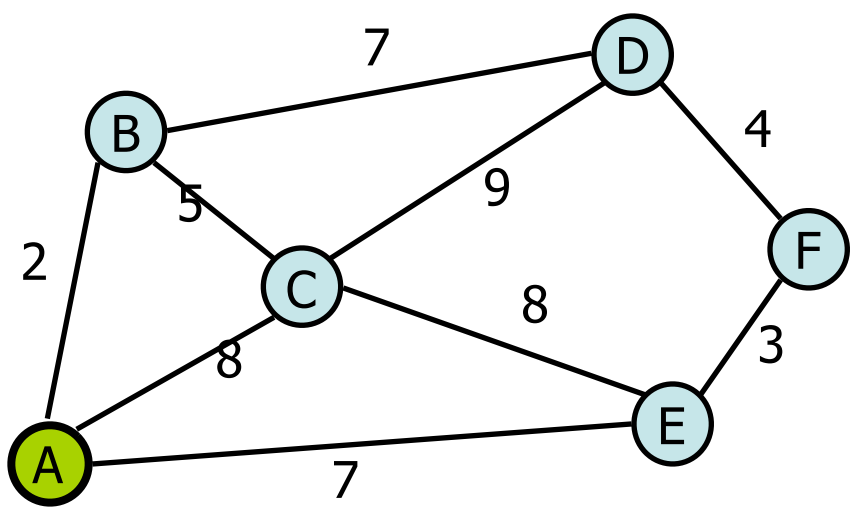
Theorem suggests an algorithm...



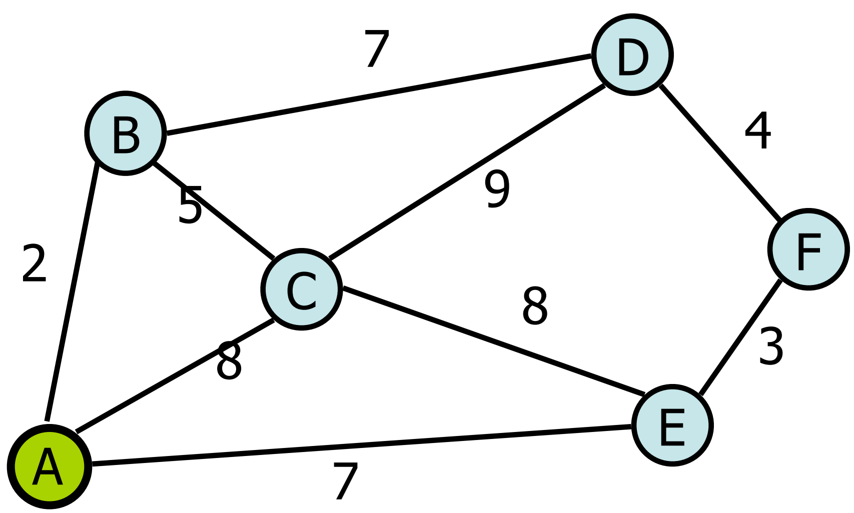
Example of Prim's algorithm -

Initialize structure:

1. For all v, d[v] = "infinity", p[v] = null
2. Initialize source: d[s] = 0
3. Initialize priority (min) queue
4. Initialize set of labeled vertices to ∅.



Example of Prim's algorithm -



Initialize structure:

1. For all v, d[v] = "infinity", p[v] = null
2. Initialize source: d[s] = 0
3. Initialize priority (min) queue
4. Initialize set of labeled vertices to ∅.

Repeat these steps n times:

- Find & remove minimum d[] unlabelled vertex: v
- Label vertex v
- For all unlabelled neighbors w of v,

  If cost(v,w) < d[w]

  d[w] = cost(v,w)

  p[w] = v

Prim's Algorithm (undirected graph with unconstrained edge weights):

Initialize structure:

1. For all v, d[v] = "infinity", p[v] = null
2. Initialize source: d[s] = 0
3. Initialize priority (min) queue
4. Initialize set of labeled vertices to ∅.

Repeat these steps n times:

- Remove minimum d[] unlabeled vertex: v
- Label vertex v (set a flag)

- For all unlabeled neighbors w of v,

  If cost(v,w) < d[w]

    d[w] = cost(v,w)

    p[w] = v

	adj mtx	adj list
heap	$O(n^2 + m \log n)$	$O(n \log n + m \log n)$
Unsorted array	$O(n^2)$	$O(n^2)$

Which is best?

Depends on density of the graph:

Sparse

Dense

Single source shortest path

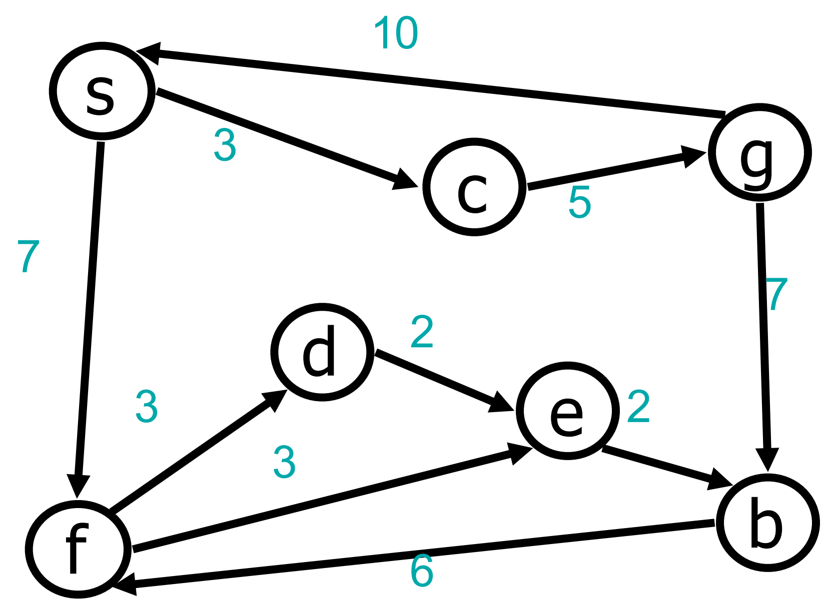
Given a start vertex (source) s, find the path of least total cost from s to every vertex in the graph.

Single source shortest path:

Input: directed graph G with non-negative edge weights, and a start vertex s.

Output: A subgraph G' consisting of the shortest (minimum total cost) paths from s to every other vertex in the graph.

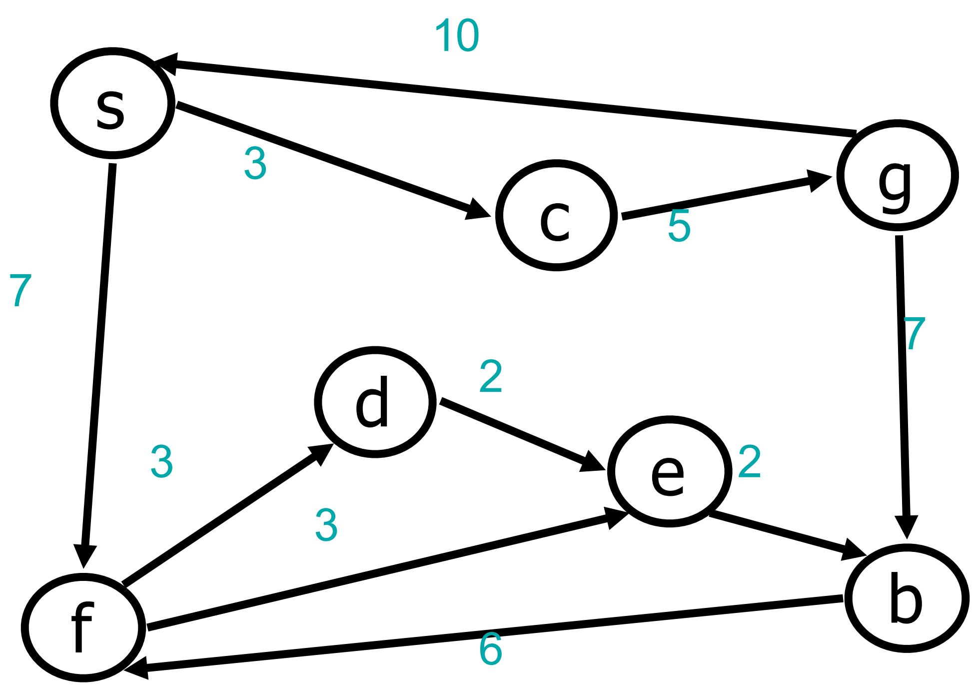
Dijkstra's Algorithm (1959)



Single source shortest path (directed graph w non-negative edge weights):

Dijkstra's Algorithm (1959)

Given a source vertex s, we wish to find the shortest path from s to every other vertex in the graph.



Initialize structure:

Repeat these steps:

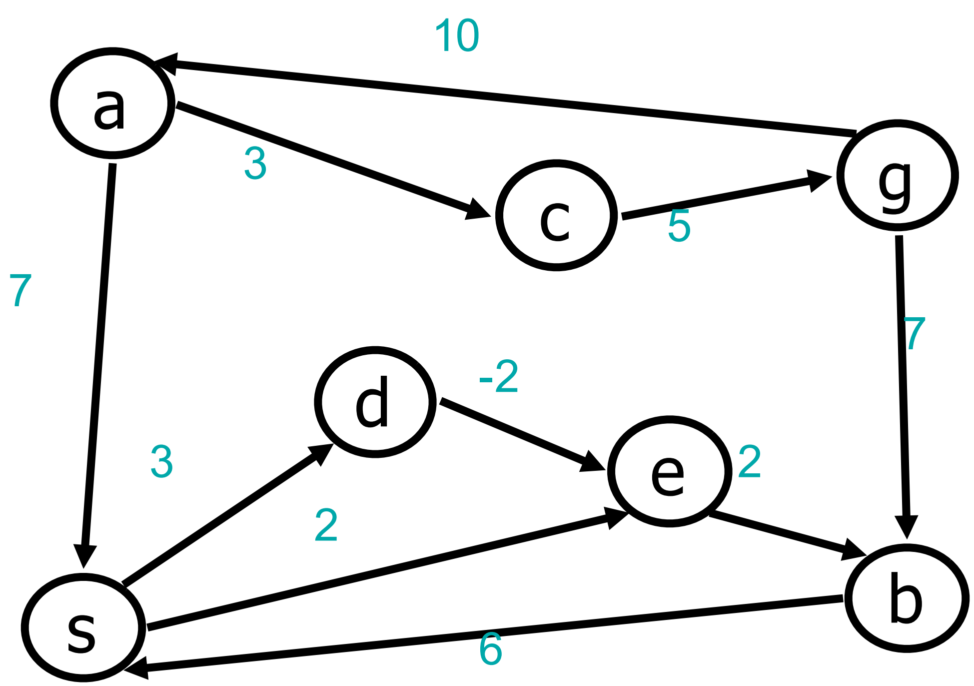
1. Label a new (unlabelled) vertex v, whose shortest distance has been found

2. Update v's neighbors with an improved distance

Single source shortest path (directed graph w non-negative edge weights):

Dijkstra's Algorithm (1959)

Why non-negative edge weights??



Initialize structure:

Repeat these steps:

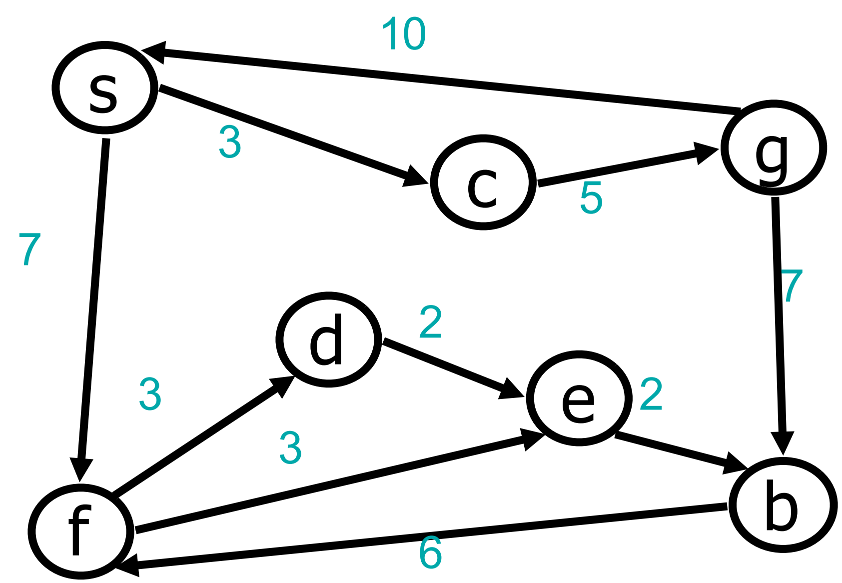
1. Label a new (unlabelled) vertex v, whose shortest distance has been found

2. Update v's neighbors with an improved distance

Single source shortest path (directed graph w non-negative edge weights):

Initialize structure:

1. For all v, d[v] = “infinity”, p[v] = null
2. Initialize source: d[s] = 0
3. Initialize priority (min) queue



Repeat these steps n times:

- Find minimum d[] unlabelled vertex: v
- Label vertex v
- For all unlabelled neighbors w of v,

  If (_____________ < d[w])

  d[w] = ________________

  p[w] = v

Running time?