

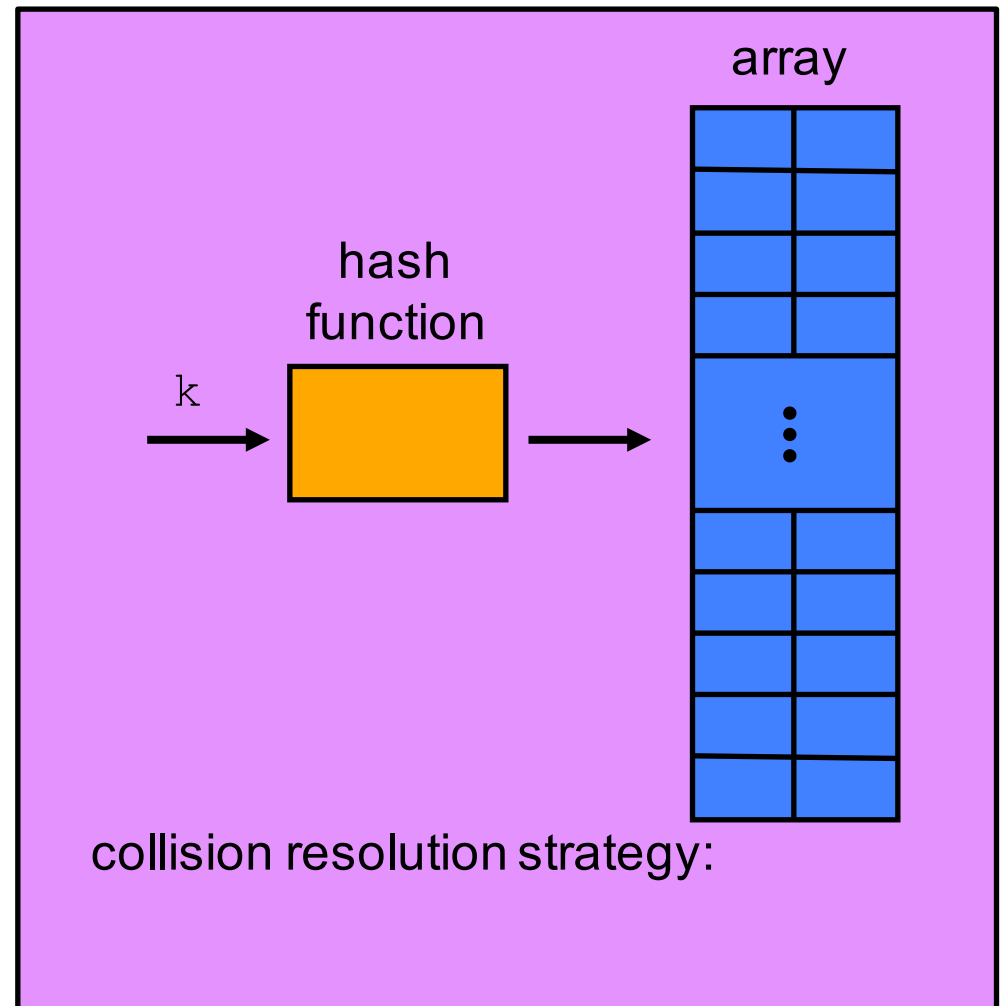
# Today's announcements:

MP6 available, due 11/17, 11:59p. EC due 11/10, 11:59p

client code

declares an object  
of ADT dictionary  
`dict<ktype, vtype> d;`  
  
ex: insert is `d[k] = v;`

class `dict`

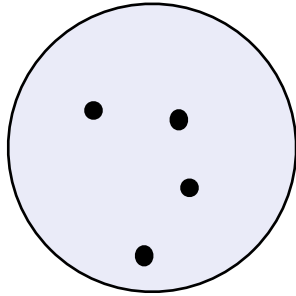


# Hash Functions:

- Consist of 2 parts:
  - A Hash: Function mapping a key to an integer  $i$
  - A compression: function mapping  $i$  into the array cells 0 to  $N-1$ .
- Choosing a hash function is tricky...
  - Don't create your own (yet)
  - Smart people can produce poor hash functions (what's a bad hash function?)
    - Knuth's multiplicative hash in "the Art of Computer Programming"
- Characteristics:
  - Computed in \_\_\_\_\_ time.
  - Deterministic.
  - Satisfy the SUHA.

# Hash Functions

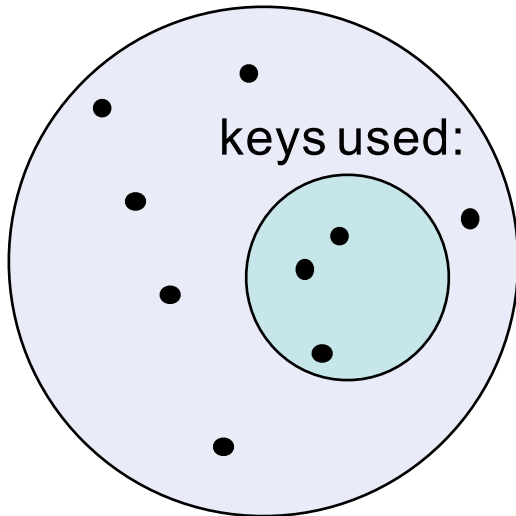
KeySpace



|   |   |  |     |  |  |  |     |
|---|---|--|-----|--|--|--|-----|
| 0 | 1 |  | ... |  |  |  | N-1 |
|   |   |  |     |  |  |  |     |

Easy, if  
 $|\text{KeySpace}| \sim N$

KeySpace



|   |   |  |     |  |  |  |     |
|---|---|--|-----|--|--|--|-----|
| 0 | 1 |  | ... |  |  |  | N-1 |
|   |   |  |     |  |  |  |     |

# Hashing Strings (an example)

Given: 8 character strings are easy to hash

The idea: Select 8 random positions from long strings and hash that substring.

A bunch of strings:

Lookyhere, Huck, being rich ain't going  
No! Oh, good-licks; are you in real dead  
Just as dead earnest as I'm sitting here  
nto the gang if you ain't respectable, y  
Can't let me in, Tom? Didn't you let me  
Yes, but that's different. A robber is m  
irate is -- as a general thing. In most  
Now, Tom, hain't you always ben friendly  
ut, would you, Tom? You wouldn't do that  
Huck, I wouldn't want to, and I DON'T wa  
ay? Why, they'd say, 'Mph! Tom Sawyer's  
t!' They'd mean you, Huck. You wouldn't  
uck was silent for some time, engaged in  
Well, I'll go back to the widder for a m  
can come to stand it, if you'll let me  
All right, Huck, it's a whiz! Come along  
Will you, Tom -- now will you? That's go  
he roughest things, I'll smoke private a  
hrough or bust. When you going to start  
Oh, right off. We'll get the boys togeth

# Hashing Strings (an example)

Given: 8 character strings are easy to hash

The idea: Select 8 random positions from long strings and hash that substring.

A bunch of strings:

```
http://en.wikipedia.org/wiki/Le%C5%9Bna_Grobla
http://en.wikipedia.org/wiki/Blow_the_Man_Down
http://en.wikipedia.org/wiki/Swen_K%C3%B6nig
http://en.wikipedia.org/wiki/2/7th_Cavalry_Commando_Regiment_(Australia)
http://en.wikipedia.org/wiki/Salman_Ebrahim_Mohamed_Ali_Al_Khalifa
http://en.wikipedia.org/wiki/Alice_High_School
http://en.wikipedia.org/wiki/Beautiful,_Dirty,_Rich
http://en.wikipedia.org/wiki/RFA_Sir_Bedivere_(L3004)
http://en.wikipedia.org/wiki/Birthright_(band)
http://en.wikipedia.org/wiki/Jacky_Vimond
http://en.wikipedia.org/wiki/Vachon
http://en.wikipedia.org/wiki/McCarthy_%26_Stone
http://en.wikipedia.org/wiki/Salisbury,_New_Hampshire
http://en.wikipedia.org/wiki/A_Line_of_Deathless_Kings
http://en.wikipedia.org/wiki/Newfoundland_Irish
http://en.wikipedia.org/wiki/Beatrice_Politi
http://en.wikipedia.org/wiki/Bona_Sijabat
http://en.wikipedia.org/wiki/Sour_sanding
http://en.wikipedia.org/wiki/Dr_Manmohan_Singh_Scholarship
http://en.wikipedia.org/wiki/Religion_in_Jordan
```

## Collision handling - Separate Chaining: (an example of open hashing)

$S = \{16, 8, 4, 13, 29, 11, 22\}$

$|S| = n$        $h(k) = k\%7$

|   |  |
|---|--|
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

|             | <b>Worst case</b> | <b>Under SUHA</b> |
|-------------|-------------------|-------------------|
| Insert      |                   |                   |
| Remove/find |                   |                   |

## Collision Handling - Probe based hashing: (example of closed hashing)

$S = \{16, 8, 4, 13, 29, 11, 22\}$

$|S| = n$        $h(k) = k \% 7$

|   |  |
|---|--|
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

Try  $h(k) = (k + 0) \% 7$ . If full...

try  $h(k) = (k + 1) \% 7$ . If full...

try  $h(k) = (k + 2) \% 7$ . If full...

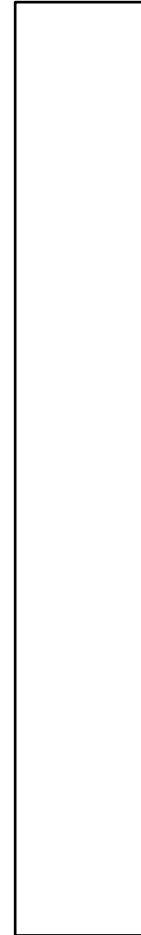
try...

## Probe based hashing – 2 problems...

Removals:

|   |    |
|---|----|
| 0 | 22 |
| 1 | 8  |
| 2 | 16 |
| 3 | 29 |
| 4 | 4  |
| 5 | 11 |
| 6 | 13 |

Clustering:





## Probe based hashing: (double hashing)

$S = \{16, 8, 4, 13, 29, 11, 22\}$      $|S| = n$      $H(k,i) = h_1(k) + ih_2(k)$

|   |  |
|---|--|
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

Try  $h(k) = (k + 0 \cdot h_2(k)) \% 7$ . If full...  
try  $h(k) = (k + 1 \cdot h_2(k)) \% 7$ . If full...  
try  $h(k) = (k + 2 \cdot h_2(k)) \% 7$ . If full...  
try...

## Hash table performance: expected # of probes for Find(key) under SUHA

Linear probing -

successful:  $\frac{1}{2} (1 + 1/(1-\alpha))$

unsuccessful:  $\frac{1}{2} (1 + 1/(1-\alpha))^2$

Double hashing -

successful:  $1/\alpha \ln 1/(1-\alpha)$

unsuccessful:  $1/(1-\alpha)$

Separate chaining -

successful:  $1 + \alpha/2$

unsuccessful:  $1 + \alpha$

Do not memorize these!

Observe:

- As  $\alpha$  increases, running times increase...
- If  $\alpha$  is held constant then running times are constant...

Which is better?

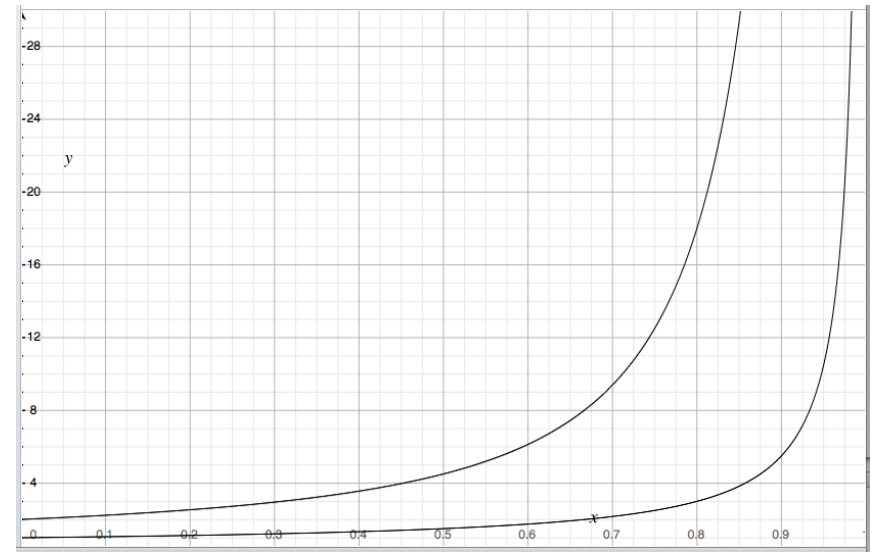
- Big records –
- Structure speed –

# Hash table performance: expected # of probes for Find(key) under SUHA

Linear probing -

successful:  $\frac{1}{2} (1 + 1/(1-\alpha))$

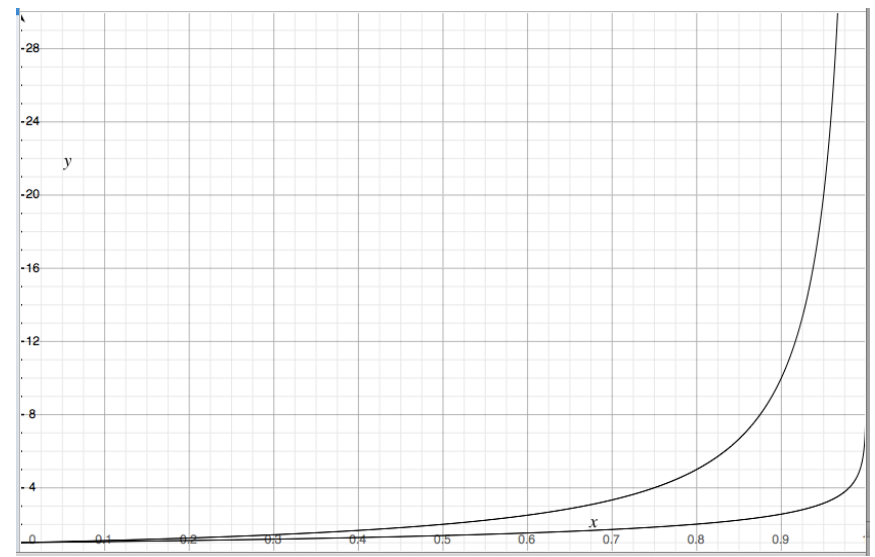
unsuccessful:  $\frac{1}{2} (1 + 1/(1-\alpha))^2$



Double hashing -

successful:  $1/\alpha \ln 1/(1-\alpha)$

unsuccessful:  $1/(1-\alpha)$



# What's left???

Running times of dictionary algorithms are a function of load factor, \_\_\_\_\_, but we hoped for \_\_\_\_\_ running times.

hmmmm....

What structures do hash tables replace for us?